



NatEquity Knowledge Base

What is the Longevity Cost Calculator and How Does It Add Value?

1 EXECUTIVE SUMMARY

Origins and History

In 1980, the U.S. Government assembled a group of medical and applied math specialists to develop a questionnaire to assess seniors in an upcoming study called the National Long-Term Care Survey (NLTC). The group spent a year deciding upon the makeup of the question set to evaluate medical, activity of daily living (ADL), cognitive and lifestyle characteristics of seniors age 65 and older. This assessment surveyed, at random, 32,000 seniors every five years for 20-years. These consecutive assessments of the same individuals were stored on computers.

After the third consecutive assessment, the actuary/demographer on the team, Eric Stallard from Duke University's Medical School, noticed some patterns in the consecutive assessment data sets. Stallard embarked on a 10-year process of developing algorithms to isolate patterns in the more than 30 million co-dependent variables in each data set. Individual snapshot data sets then had to be reconciled to changing data from consecutive 5-year interval data for each individual studied. In total 20,000 individuals were tracked until death in three or more consecutive data sets to validate the trajectory of those changes. The algorithm set in the model is more than 96% accurate in predicting the trajectory of morbidity and mortality in single individuals age 65 and older.

Stallard first presented his peer reviewed findings at a conference in 2005. The material was again peer reviewed and published in 2007ⁱ. In 2008 Stallard was presented with the Edward Lew Award, given annually to one actuary worldwide for work in computer modeling. In 2007, Life Settlement Financial, LLC (LSF)ⁱⁱ, (NatEquity's CEO, Peter Mazonas, is also the Managing Member of LSF), bought the commercial rights to the model. LSF used the original code to develop and validate a web based replica of the model and named it the Longevity Cost Calculator (LCC). NatEquity has an exclusive use license to the LCC for private jumbo reverse mortgage type products. Stallard continues to use and revalidate the model in medical research. Most recently in 2014 and again in 2017, the original study accuracy has been replicated in peer reviewed published journal articles, to accurately predict the trajectory toward death of persons with Alzheimer's Disease^{iiiiv}

Value Added, Part 1 – Superior Control of Portfolio Tail Risk

NatEquity adds value to work it does for clients by using the LCC model to select seniors with appropriate life expectancy characteristics to meet each client's objectives. For shared equity option

contract funders NatEquity exploits the LCC's ability to isolate individuals that will have longer life expectancy than predicted by commercial life expectancy underwriters using only medical records to estimate a person's life expectancy. NatEquity's proprietary knowledge allows NatEquity to exclude, should the Company chose, these homeowners from portfolios. Controlling contract duration eliminates the "tail risk" of a portfolio and stabilized the internal rate of return (IRR). Alternatively, the contracts of homeowners with longer life expectancies can be tranching for longer-term portfolios.

Value Added, Part 2 – Accretive Portfolio Values

In 2014, accounting rules worldwide implemented changes to how assets and liabilities are priced for audited financial statements. These changes, initially begun in the 1990s, require that assets be "Marked-to-Fair Value" and no longer carried at cost on financial statements. This is easy for stocks and bonds where the daily traded value is published. It is more difficult to fair value conventional forward mortgages or derivatives. Most difficult are Level-Three assets where the value is dependent upon an "unknowable" future event, moveout or death. This is where the LCC's predictive abilities, combined with a companion valuation methodology yields value^v. The ability to use Bayesian Inference to have statistical confidence in predicting future option contract portfolio cash flows allows NatEquity, under mark-to-fair value GAAP rules, to use a lower discount rate to arrive at the net present value (NPV) of future cash flows – the measure of value. This means NatEquity option contract or private jumbo reverse mortgage portfolios aggregated using the LCC accrete substantial additional annual book profits and cumulative balance sheet value, another source of leverage available to supplement reserves.

Value Added, Part 3 – First Mover Barrier to Entry

Others, most notably the Blackstone Group, have tried to enter the market to offer private jumbo reverse mortgages. The individual product was not hard to design, but when they were pressed to value a portfolio for GAAP purposes the results for unfavorable. Unable to demonstrate a viable "mark-to-fair value" methodology they and others had to fall back on the much higher discount rates required for "mark-to-fair market" valuation.

2 LONGEVITY COST CALCULATOR (LCC) - MODEL DETAIL

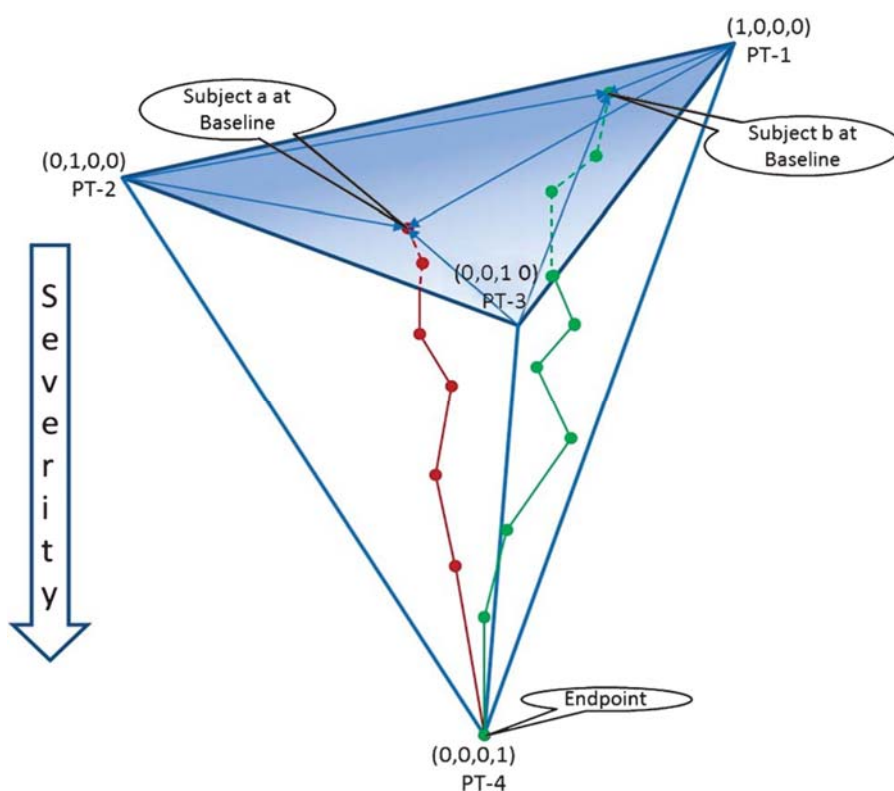
To deal with the more than 30 million codependent variable large data sets Stallard employed the Grade of Membership (GoM) model KG Manton^{vi} and MA Woodbury and Stallard had developed at Duke's Center for Demographic Studies. The GoM model does this by estimating, using maximum likelihood principles, two types of parameters. One describes the probability that a person who is exactly like one of the K analytically defined types has a particular response on a given variable. The second describes each individual's degree of membership in each of the K types. This "partial" membership score reflects the logic of the fuzzy partitions (rather than of discrete groups) that are employed in the analyses. This predictive modeling is an application of the Bayes Theorem where a hypothesis is backed by strong evidence over time. Once the algorithms are developed and validated they can be applied to single data sets. By modifying the probability structure of the basic model, the procedure can be applied to a number of different types of data and analytic problems, as evidenced in the Alzheimer's work.

Each GoM state is described in relation to an inquiry, ideal individuals "Pure Type" score totaling 1.0. GoM scores are not crisp, they are "fuzzy". Except for rare pure type individuals, an individual's score is

a blend of scores from GoM categories 1-3, where a pure type 1.0 GoM 4 score occurs at death (endpoint). GoM scores however are not probabilistic of membership – instead are grades of membership.

The GoM score categories are defined below:

1. GoM I – Generally healthy with lowest level of impairments.
2. GoM II – Poorest subjective health, largest number of medical conditions, non-institutionalized, low mortality. These are persons who will usually outlive both longevity table mortality and mortality or life expectancy derived from medical records only commercial life expectancy underwriters.
3. GoM III – High mortality rates, few medical conditions, few impairments relatively good subjective health. These are persons with activity of daily living and cognitive impairments typically not recorded on their medical records.
4. GoM IV – High mortality rates, high levels of physical and cognitive disability and institutionalization.



GoM 1 (also referred to as “Pure Type I” or “PT I” in the above drawing, with Roman numerals designating the rank ordering of the states by health status; we use the shorter GoM 1–4 reference throughout this document) refers to the healthiest component of the population. GoM scores 2–4 captures a range of health problems that occur at different ages, with progressive and graded transitions from GoM 2 to GoM 3 and 4. GoM 2 scores refer to people who have numerous medical problems, but few, if any ADL or other functional problems, or cognitive impairment. Persons with initial strong scores (i.e., close to 1.0, or 100%) on GoM 2 will live longer than traditional LE providers estimate; although this changes at older ages where these persons transition to strong scores on GoM 4.

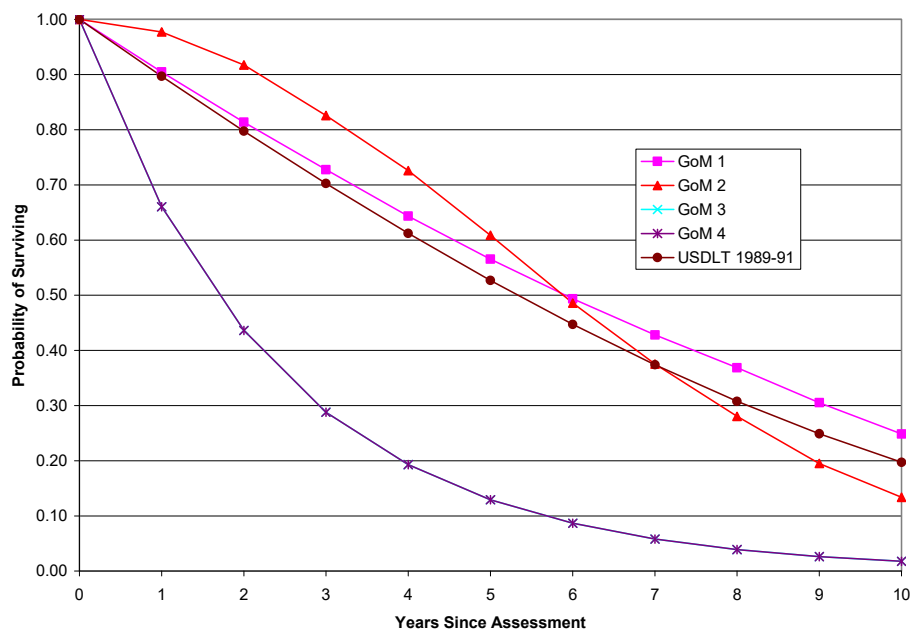
Persons with strong scores on GoM 3 have minor medical problems, but mild/moderate cognitive impairments, usually not indicated in their medical records, although this also changes at older ages where these persons transition to strong scores on GoM 4. Strong scores on GoM 4 identify people who have more serious medical problems, combined with serious ADL and/or cognitive impairments, usually not indicated in their medical records. People with initial strong scores on GoM 3 and GoM 4 will have shorter LE's than those issued by traditional LE underwriters.

Recall that the LE is the area under the relevant survival curve for the person or population for which the LE is being calculated. Thus, differences in LE between persons or groups of persons are best understood by examining the associated survival curves. This is illustrated in the following graph (Fig. 1) which plots the survival curves for males assessed at age 82 (i.e., age at last birthday is 82) for the next 10 years following the assessment, with a separate curve shown for each of the four time-invariant GoM pure types and also for comparison the survival curve from the U.S. Decennial Life Table (USDLT) for 1989-91.

Whether the USDLT or Society of Actuaries Variable Basic Tables (VBT) are used for comparisons makes little difference because the LCC is not table based. The changes in the VBT tables from 2001 through 2014 extrapolate an extension of the tail of mortality tail with a corresponding adjustment to early mortalities.

Figure 1

Probability of Surviving for Each of 10 Years Following Initial Assessment at Age 82, Males, by Initial Time-Invariant GoM Pure Type and for the U.S. Decennial Life Table for 1989-91



By age 82, the GoM 3 and GoM 4 curves have merged.

According to the USDLT, the male LE at age 82 is 6.2 years. This value is less than the LEs of 6.9 and 6.3 years for GoM 1 and 2, but is substantially higher than the LEs of 2.5 years each for GoM 3 and 4 (the values are the same because an initial GoM 3 “converts” to GoM 4 prior to age 82).

Importantly, given that both the survival curve and its slope are used to price contracts, it is important to accurately estimate these quantities. Armed with this knowledge, an underwriter can accurately price each contract and the valuation team can accurately value the NPV of future portfolio cash flows. This predictability of future cash flows puts NatEquity in a uniquely favorable market position.

On the above graph, you will note that an initial GoM 2 has approximately a 16% greater likelihood of surviving through years 1 through 5 until the lines converge at the year 6, near the 6.2 year LE. On the other hand, an initial GoM 3 or 4 has approximately a 75% lower likelihood of surviving through years 1 through 5, with corresponding reductions for persons who have initial *fractional* scores on GoM 3 or 4, with complementary *fractional* scores on GoM 1 and/or 2. The sum of all fractional scores must equal 1.0 (100%), with the scores for any given individual derived from his/her answers to the 76 questions on the application form.

Below are a table (Table 1) and two graphs (Figs. 3–4) reproduced from the 2007 NAAJ paper. Table 1 shows the predicted probability of death within each year contrasted to the observed death of individuals in the assessment population. These are shown in age brackets by the 5-year age groups (age at start of each 1-year follow-up) used for model estimation as well as the total over age of the observed vs predicted mortality for males and females.

Table 1
Probabilities of Death within One Year in Four Pure-Type GoM Model, Adjusted for Declines in Vitality, by Sex and Attained Age at Time of Exposure

Exposure Age	No. of Person-Years at Risk ¹	Annual Probability by Type				Observed Probability	Predicted Probability
		I	II	III	IV		
Males							
65–69	20,323	0.000	0.000	0.132	0.138	0.031	0.032
70–74	38,255	0.002	0.005	0.194	0.246	0.043	0.044
75–79	31,291	0.038	0.013	0.319	0.319	0.067	0.067
80–84	19,170	0.095	0.023	0.340	0.340	0.105	0.106
85–89	8,117	0.127	0.202	0.330	0.330	0.154	0.155
90–94	2,728	0.198	0.323	0.323	0.323	0.228	0.229
95–99	793	0.226	0.434	0.434	0.414	0.301	0.301
100–104	155	0.372	0.528	0.528	0.528	0.400	0.401
Total	120,832	0.041	0.033	0.253	0.271	0.071	0.072
Females							
65–69	25,424	0.000	0.000	0.081	0.140	0.017	0.017
70–74	52,008	0.001	0.003	0.108	0.223	0.027	0.027
75–79	48,498	0.018	0.003	0.249	0.249	0.043	0.043
80–84	35,563	0.059	0.005	0.267	0.267	0.070	0.070
85–89	20,404	0.089	0.110	0.271	0.271	0.115	0.115
90–94	9,577	0.127	0.272	0.272	0.272	0.183	0.184
95–99	3,804	0.168	0.388	0.388	0.388	0.264	0.264
100–104	992	0.274	0.499	0.499	0.499	0.325	0.324
Total	196,270	0.036	0.037	0.201	0.239	0.060	0.061

¹Includes up to four observations per respondent; excludes respondents aged 65–69 in 1999.
Source: Author’s calculations based on data from the NLTCs.

The differences between the observed and predicted age-specific death probabilities are very small and are statistically nonsignificant, with chi-squared values of 1.36 and 0.29, respectively, each with 6 d.f. (reference values are 12.59 and 16.81 at the conventional 5% and 1% significance levels). The graphs show the same data but add the corresponding death probabilities from the U.S. Decennial Life Table (USDLT) for 1989-91.

Figure 3
Probability of Death within One Year, Males

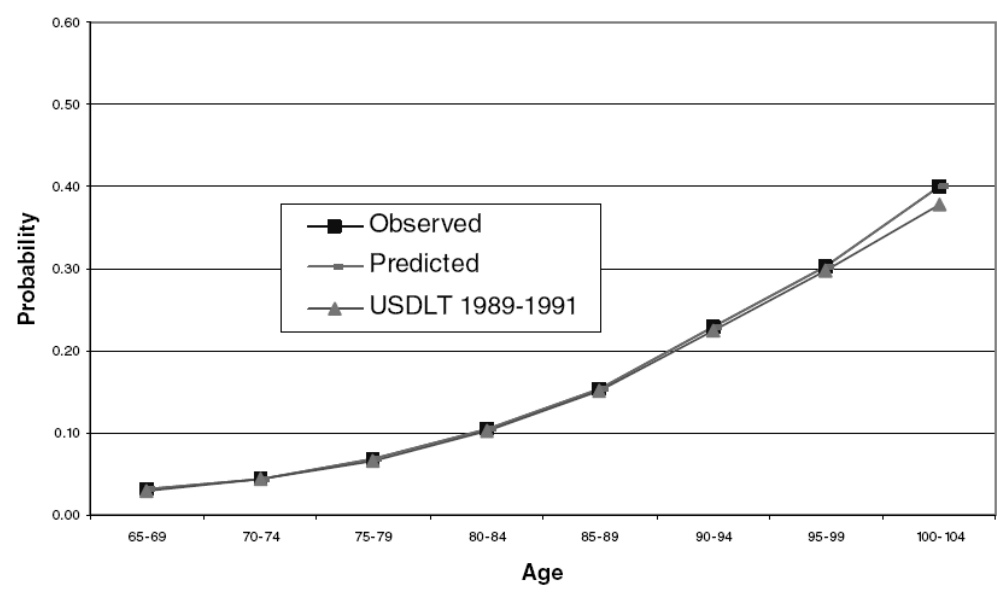
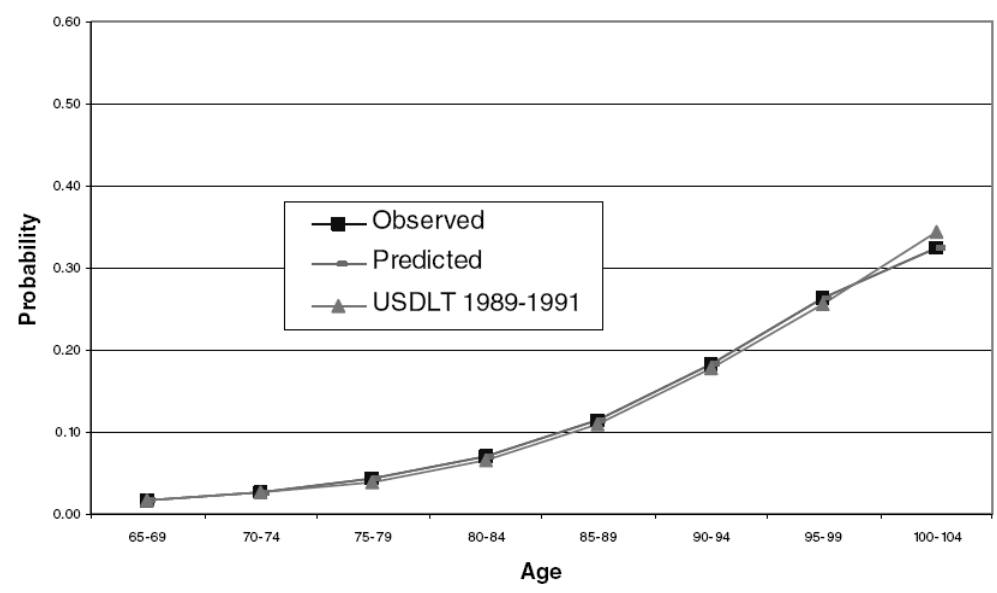


Figure 4
Probability of Death within One Year, Females



Below we attempt to translate these values into the numbers presented by commercial LE providers who represent that they are accurate 86% to 96% of the time. They do not provide nor publish the basis for their estimates. Hence, we cannot be certain that our measures of accuracy are fully comparable with theirs. We do know from a 2010 study done for Deutsche Bank, where 4 independent life expectancy reports were secured from each individual's medical records, that the difference between the low and the high LE for each of 500+ persons, had a delta of 30%. From the same data, when the difference between the high and low LE was more than 30%, the delta was 57%^{vii}.

To begin consideration of measures of accuracy, it is useful to consider the random statistical fluctuations that are expected from estimates based on the different numbers of events likely to be observed in different sized samples, assuming fixed underlying event rates. The table below shows the minimum number of expected events needed to cap the maximum relative error (e.g., [observed number of deaths minus expected number of deaths] ÷ [expected number of deaths]) with one of three levels of confidence.

Table 2
Credibility and Event Counts

Maximum Acceptable Departure from the Expected Count	Probability of Observed Count Falling Within the Acceptable Range		
	90%	95%	99%
	Minimum Required Expected Count		
+/-2.5%	4,329	6,146	10,616
+/-5.0%	1,082	1,537	2,654
+/-7.5%	481	683	1,180
+/-10%	271	384	663
+/-20%	68	96	166
+/-30%	30	43	74
+/-40%	17	24	41
+/-50%	11	15	27

Source: Based on Longley-Cook (1962).

To be 99% confident that the maximum relative error is less than 2.5%, the sample size needs to be large enough to produce 10,616 deaths (in bold in the table). The annual mortality probabilities in the *NAAJ* analysis were based on 20,428 deaths (8,583 males and 11,845 females), indicating that the total rates are very stable but the stratifications by age and other variables may be affected by random statistical fluctuations. To be 90% confident that the maximum relative error is less than 5%, the sample size needs to be large enough to produce 1,082 deaths, which is the standard size used for full credibility in the actuarial literature. To be 95% confident that the maximum relative error is less than 20%, the sample size needs to be large enough to produce 96 deaths, which rounds to about 100. To be 90% confident that the maximum relative error is less than 50%, the sample size needs to be large enough to

produce 11 deaths, which rounds to about 10. Thus, as the expected number of deaths falls from 10,000 to 1,000 to 100 to 10, the relative error increases from about 2.5% to 50%.

Practical considerations often dictate sample sizes less than that needed for full actuarial credibility. Table 2 indicates that sample sizes of 271 and 384 can yield relative errors of $\pm 10\%$ at the 90% and 95% confidence levels, respectively, consistent with A.M. Best's recommendation that the collateral pool for longevity dependent life settlement portfolios consist of at least 300 lives.

Random statistical fluctuations are inherently unpredictable. Hence, our measures of accuracy must focus on our ability to generate accurate values for the expected number of deaths among any selected set of insured lives.

The tables and graphs from the *NAAJ* paper (Table 1; Figs. 3–4) show that this can be done for groups of insured lives when the groups are defined on the basis of age and sex.

The next two graphs (Figs. 5–6) show the performance of the model when the NLTC mortality-exposure data are grouped into 10 categories according to the predicted probability of death, based on the use of fixed cutpoints at multiples of .05 (5%), separately for males and females. Chi-squared statistical tests of fit of the models are presented separately in Tables 3–4.

Figure 5

Observed and Predicted Probabilities of Death, Males, by Predicted-Probability Class Intervals with Cutpoints at Multiples of 5%

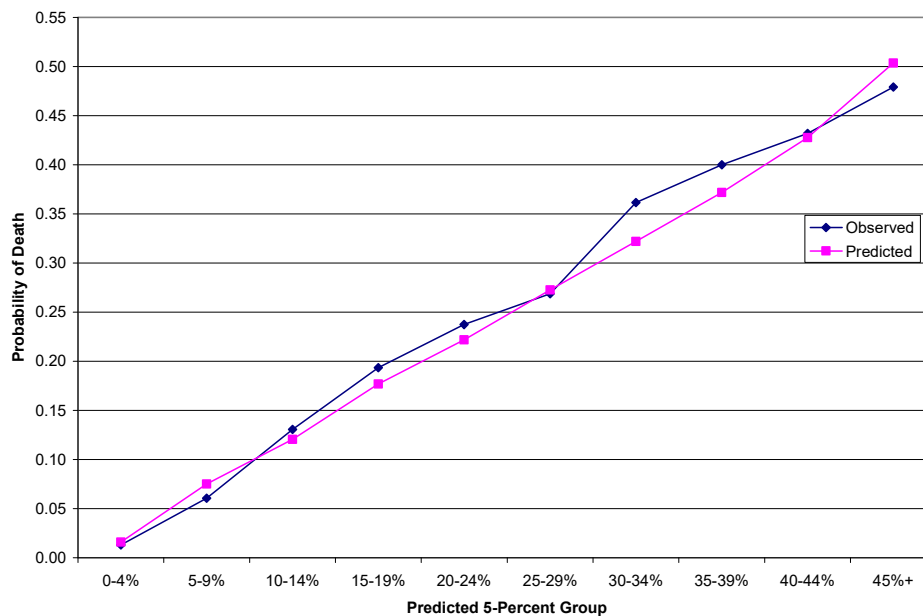
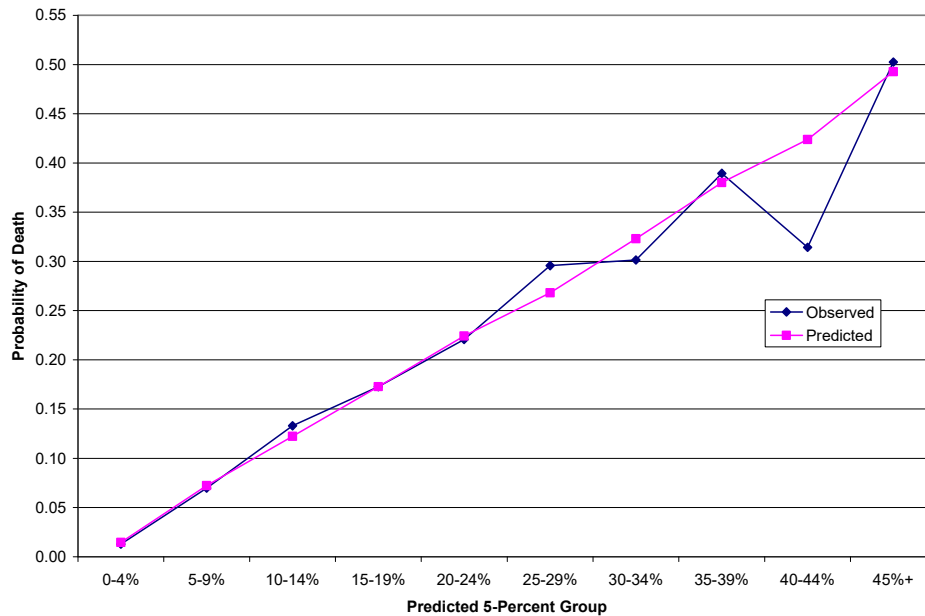


Figure 6

Observed and Predicted Probabilities of Death, Females, by Predicted-Probability Class Intervals with Cutpoints at Multiples of 5%



Visually, one can see that the observed probabilities increase across the 10 categories for both sexes, except for category 9 for females. However, the chi-square test of the deviation for that one point indicates that the difference is statistically nonsignificant, with a chi-squared value of 3.44 with 1 df (Table 4; reference values are 3.84 and 6.63 at the conventional 5% and 1% significance levels).

Table 3

Observed and Predicted Probabilities of Death, Males, by Predicted-Probability Class Intervals with Cutpoints at Multiples of 5%

Percentage Group	Number of Person-Years at Risk	Observed Number of Deaths	Expected Number of Deaths	Observed Probability	Predicted Probability	Hosmer-Lemeshow Chi-Squared
0-4%	61,463	801	981	0.013	0.016	33.60
5-9%	23,256	1,407	1,747	0.061	0.075	71.73
10-14%	20,100	2,622	2,420	0.130	0.120	19.20
15-19%	7,705	1,490	1,363	0.193	0.177	14.29
20-24%	4,557	1,082	1,011	0.237	0.222	6.44
25-29%	2,095	563	571	0.269	0.273	0.16
30-34%	1,361	492	438	0.361	0.322	9.73
35-39%	115	46	43	0.400	0.372	0.39
40-44%	132	57	56	0.432	0.428	0.01
45%+	48	23	24	0.479	0.504	0.11
Total	120,832	8,583	8,655	0.071	0.072	155.65

Table 4
Observed and Predicted Probabilities of Death, Females, by Predicted-Probability Class
Intervals with Cutpoints at Multiples of 5%

Percentage Group	Number of Person-Years at Risk	Observed Number of Deaths	Expected Number of Deaths	Observed Probability	Predicted Probability	Hosmer-Lemeshow Chi-Squared
0-4%	111,425	1,434	1,614	0.013	0.014	20.33
5-9%	44,837	3,124	3,239	0.070	0.072	4.42
10-14%	18,004	2,396	2,202	0.133	0.122	19.46
15-19%	10,245	1,770	1,770	0.173	0.173	0.00
20-24%	6,536	1,443	1,465	0.221	0.224	0.43
25-29%	3,413	1,009	915	0.296	0.268	13.17
30-34%	617	186	199	0.301	0.323	1.32
35-39%	914	356	347	0.389	0.380	0.35
40-44%	70	22	30	0.314	0.424	3.44
45%+	209	105	103	0.502	0.493	0.08
Total	196,270	11,845	11,885	0.060	0.061	62.99

The Hosmer-Lemeshow chi-squared test produces statistically significant total chi-squared values of 155.65 and 62.99, respectively, for males and females, each with 8 d.f. (reference values are 15.51 and 20.09 at the conventional 5% and 1% significance levels). These corroborate the better than 96% accuracy of the model generated by linear regression.

Several comments are in order:

1. The tests indicate that the models displayed in Figures 5 and 6 do *not* fit the data. This means that at least some of the deviations of the observed from predicted numbers of deaths are larger than expected by chance.
2. These are identified by the boldface font in the rightmost columns of Tables 3 and 4 using a cutpoint equal to the critical value of 6.63 using the conventional 1% significance level.
3. For males, the five significant deviations are for the four lowest probability groups 0–19% and 30–34%; for females, the three significant deviations are for the 0–4%, 10–14%, and 25–29% probability groups.
4. The expected counts for the eight groups with significant deviations ranged from 438 to 2,420, and five of the eight exceed the 1,082 cutpoint for highly credible data in Table 2; all eight exceed the minimum level of 300 lives recommended by A.M. Best for life settlement collateral pool sizes.

Practical Application – Portfolio Valuation

In 2009 the Longevity Cost Calculator was presented to a panel at the Securities and Exchange Commission as a component of a methodology to value Level 3 longevity dependent assets where the outcome is unknowable. This methodology was published in 2011^{viii}. It is today widely referenced in academic and accounting journals.

This valuation methodology uses the combination of three commercial medical records only life expectancy reports and the output of the LCC model. At the outset the Bayesian inference “priors” assume all four life expectancy reports have equal weightings of .25 each. The hypothesis is that the evidence over time will lead to the “posterior” prediction that the LCC has statistical significance and can be relied upon as the accurate predictor of future portfolio cash flows. Modeling the Bayesian theorem shows that at 100 policy maturities the LCC has achieved better than 95% statistical confidence.

For readers not familiar with Thomas Bayes’ theorem from 1750s, I recommend the New York Times Article^{ix} referenced below and two recent books: *The Signal and the Noise*, Nate Silver, Penguin Press, 2012; and *The Theory That Would Not Die: How Bayes’ Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy*, Sharon Bertsch McGrayne, Yale University Press, 2011.

3 CONCLUSION - LONGEVITY COST CALCULATOR (LCC) BRINGS STRONG VALUE ADD TO MEASURING FUTURE CASH FLOWS

Consistently applying the Bayesian Inference valuation methodology published by Graham, Stallard and Mazonas allows, after approximately 100 maturities, the predicting of future portfolio cash flows with statistical confidence. This high level of predictable confidence attained using the LCC longevity assessment permits the valuation team to use a much lower risk premium factor in the discount rate used to NPV the future portfolio cash flows. This lower risk premium, 5%, added to a 2.5% risk-free cost of capital, yields an NPV that accretes value to both the balance sheet asset and the corresponding addition of income to the book balance sheet. This is on a basis where a discount rate of approximately 11.5% produces a valuation at cost with no accreted or diminished value.

Without this predictability, auditors have been known to apply a 16% risk premium on top of a 2.5% risk free cost of capital. This yields a significant decrease in asset valuation and large reductions to book income and earnings per share value. This decrease in valuation is only earned back over time, not time that issuers of audited financial statements will tolerate.

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ⁱ Stallard, E., 2007. Trajectories of Morbidity, Disability, and Mortality among the U.S. Elderly Population: Evidence from the 1984-1999 NLTCs. *North American Actuarial Journal* 11(3):16–53.
<http://www.soa.org/library/journals/north-american-actuarial-journal/2007/july/naaj0703-2.pdf>

ⁱⁱ Life Settlement Financial, LLC, is the Delaware Limited Liability Company, licensed in 2007, that is the legal owner of the state regulated Life Settlement Provider Licenses in California and Oregon. We operate under that name in the life settlement business. In 2013, recognizing the extension of our business into reverse mortgage type products and longevity dependent asset management we registered the dba of Longevity Specialty Finance in California. LSF operates under that dba in all markets other than life settlements.

ⁱⁱⁱ *A New Algorithm for Predicting Time to Disease Endpoints in Alzheimer's Patients*, J. Alzheimer's Dis. 2014 Jan 1, 38(3), 10.3233/JAD-131142, <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3864687>

^{iv} *Personalized predictive modeling for patients with Alzheimer's disease using an extension of Sullivan's life table model*, E. Stallard, B Kinosian and Y Stern, Alzheimer's Research & Therapy (2017) 9:75.

^v *Longevity Risk in Fair Valuing Level-Three Assets in Securitized Portfolios*, Peter Macrae Mazonas, Patrick John Eric Stallard, Lynford Graham, the Geneva Papers (2011) 36, 516-543. Doi: 10.1057 / gpp 2011.25.

^{vi} <http://www.ncbi.nlm.nih.gov/pubmed/1820287>

^{vii} Unpublished analysis by Peter Mazonas of raw de-identified data provided by the firm who conducted the study and compiled the individual case files.

^{viii} <http://www.palgrave-journals.com/gpp/journal/v36/n4/abs/gpp201125a.html> Copies available upon request.

^{ix} *The Odds, Continually Updated*, F. D. Flam, New York Times, September 29, 2014.

<http://www.nytimes.com/2014/09/30/science/the-odds-continually-updated.html? r=0>